

Session I Examination 2021

BSc 3rd Semester

Subject: Mathematics (H)

Paper MAT-HC-3016: Theory of Real Function

Marks: 30, Time: 1 hour

6×5=30

Answer any six questions

1. What do you mean the continuity of a function? Show that every continuous function is differentiable but the converse is not true.
2. State Roll's theorem. Verify Roll's theorem for the function $f(x) = \sin x$, in $[0, \pi]$.
3. State and proof Lagrange's mean values theorem.
4. State and proof Cauchy's mean values theorem.
5. State Taylor's theorem. If $f''(x)$ is continuous at $x = a$ then

$$\lim_{h \rightarrow 0} \frac{f(a+h) - 2f(a) + f(a-h)}{h^2} = f''(a).$$

6. If $a < b$, prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$.

7. If $f(x) = (x-1)(x-2)(x-3)$ and $a = 0$, $b = 4$, find the value of c satisfying

$$\frac{f(b) - f(a)}{b - a} = f'(c) \text{ where } a < c < b.$$
